RLC Circuits and Resonance

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The concept of resonance is one of the most important in all of physics and engineering.

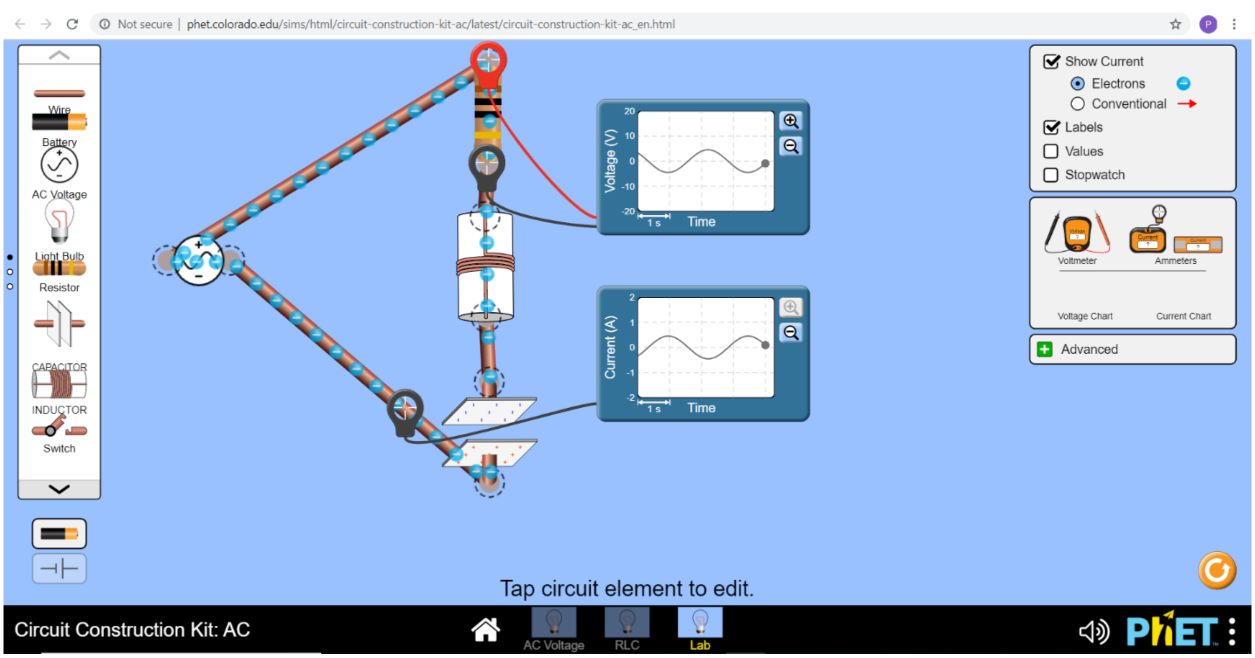
We will be studying resonance in AC circuits. But the concept has much wider application than that.

The length of an organ pipe is determined by the frequency at which the organ builder wants that pipe to give a loud sound – that is, to be resonant.

The electronics of transmitting and detecting radio waves is all about resonance. An RLC circuit – that is, a circuit with resistance, inductance and capacitance – has a resonant frequency. To detect radio waves with a particular frequency, a variable capacitor is tuned to a particular capacitance to give the circuit a resonant frequency that equals the frequency of the radio waves that a user desires to detect.

We'll explore the idea of resonance by trying to find a resonant frequency in an RLC circuit in our simulation.

Our simulation has a range of AC source frequencies (0.1-2.0 Hz), inductances (10-100 H) and capacitances (0.05-0.20 F). Your AC Circuit Cheat Sheet includes an equation for calculating the resonant frequency – and it only involves the inductance and the capacitance of the circuit (and not the resistance, which is why I didn't mention it in the previous sentence). Is there some combination of available (in our simulation) inductance and capacitance values that gives a resonant frequency available in the simulation?



To really nail the resonance, we would need more precision in selecting the frequency than is available in the simulation. We only have one decimal place of precision in choosing the frequency and we would really need two decimal places to kill it. But when you build an RLC circuit in the simulation, set your inductance and capacitance to the values you found above and then set your frequency as close as you can to the resonance frequency you calculated, what are the amplitudes of the voltage across the resistor and the current? What happens to the amplitudes of the voltage across the resistor and the current as you move the frequency away from the resonant frequency? Recall that the power output of a circuit is the product of the voltage drop and the current. So the resonant frequency – where the amplitudes of the voltage across the resistor and the current as maximums – gives the greatest power output by the resistor (which is the whole idea of resonance).

One more concept to examine – impedance. Impedance is yet another resistance-like concept. But impedance doesn't just take one circuit component into account like capacitive reactance or inductive reactance. Instead, it takes into account resistance, capacitance and inductance all at once. It can be used to calculate the amplitude of the current in an RLC circuit from the amplitude of the voltage given by the AC source.

The impedance is given by

$$Z = \sqrt{R^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2}$$

Since the inductive and capacitive reactances are frequency-dependent, the impedance is as well.

The relationship between the amplitude of the current and the amplitude of the voltage from the AC source is then

$V = IZ$

Set the frequency in the simulation to be different from the resonance frequency, and calculate the impedance at that frequency. Now click on the AC source in the simulation to find the amplitude of the voltage from the source, and read the amplitude of the current from the current chart. Does the $V = IZ$ equation work?

One more observation about the impedance: You found values of inductance and capacitance above that allowed you to (more or less) see the effect of resonance in a circuit in your simulation. If you calculate the inductive and capacitive reactances at resonance, how do they compare? What does that do to the expression for impedance given above?